Total No. of Questions – 8] [Total No. of Printed Pages – 4

BE-IV/6(A)

212804

COMPUTER ENGINEERING **COURSE NO. MTH – 413**

(Discrete Mathematics)

Time Allowed - 3 Hours *Maximum Marks -100*

Note: Atte[illegible] questions in all selecting at least two questions from each Section. Each question carries 20 marks. Use of calculator is allowed.

SECTION – A

1. (a) Prove that the relation R on the set NI of natural numbers defined by aRb $\Longleftrightarrow$ (a – b) is divisible by n, where n $\in$ NI, is an equivalence relation.

   (b) Among 100 students, 32 study mathematics, 20 study physics, 45 study biology, 15 study mathematics and biology, 7 study mathematics and physics, 10 study physics and biology and 30 do not study any of the three subjects. Find the number of students studying exactly one of the three subjects.

   (c) Show that:

   $$1.2.3 + 2.3.4 + 3.4.5 + \cdots + n(n+1)(n+2) = \frac{n(n+1)(n+2)(n+3)}{4}$$

   Using mathematical induction. (6, 7, 7)

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2. (a) Show that the set IR of real numbers is uncountable.

(b) Define a bijective map. Consider the function of $f: Q \setminus \{3\} \to Q$ define as $f(x) = \frac{2x+3}{x-3}$, for all x, where Q is the set of rational numbers. Verify whether f is a bijection or not?

(c) Define generalized pigeonhole principle. Let there are five men and three women at a function. If these people are lined up in a row, show that at least two men will be next to each other. (6, 7, 7)

3. (a) Define the following terms with an example:

(i) Simple group (ii) Abelian group

(iii) Left coset (iv) Cyclic group

(v) Homorphism of a group.

(b) Prove that every group of prime order is cyclic.

(c) Prove that the set G = { 1, 2, 3, 4, 5, 6 } forms a group under multiplication modulo 7. (10, 5, 5)

4. (a) State and prove the necessary and sufficient conditions for a non-empty subset H of a group G to be a subgroup.

(b) Prove that a subgroup H of a group G is a normal subgroup iff for every x in G, $xHx^{-1} = H$.

(c) Define integral domain. Prove that every finite integral domain is a field. (7, 6, 7)

SECTION – B

5 (a) Explain Konisberg bridge problem.

(b) Prove that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$.

(c) Define degree of a vertex. Prove that there does not exist a graph with 5 vertices with degree equal to 1,3,4,2,3 respectively. (7, 7, 6)

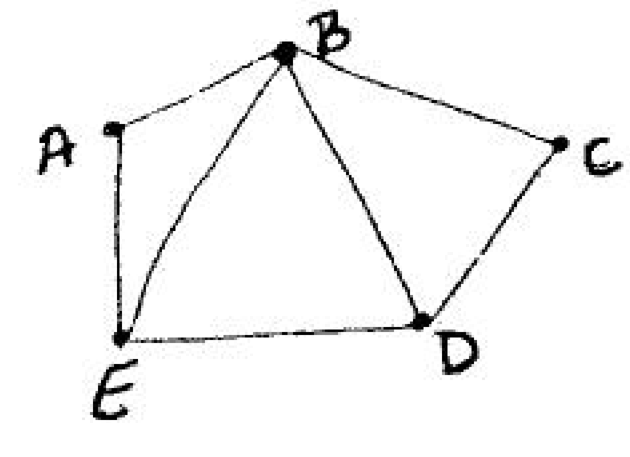
6. (a) State and prove Euler's formula for planar graph.

(b) Define Hamiltonian graph. Prove that there is always a Hamiltonian path in a directed complete graph.

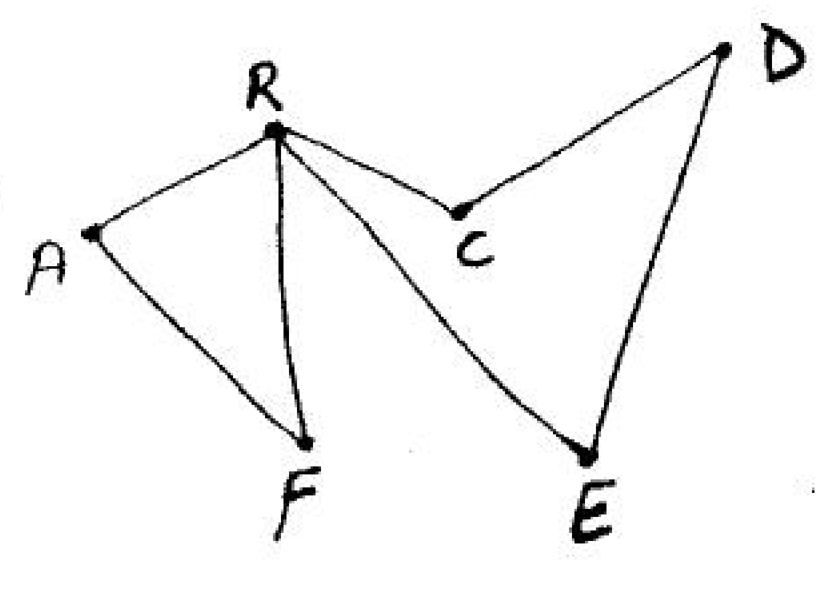
(c) Define k – regular graph. Find k, if a k-regular graph with 7 vertices has 14 edges. Also draw the k – regular graph. (7, 7, 6)

7. (a) Define spanning tree. Generate spanning trees for the graphs given below:

(i)



(ii)



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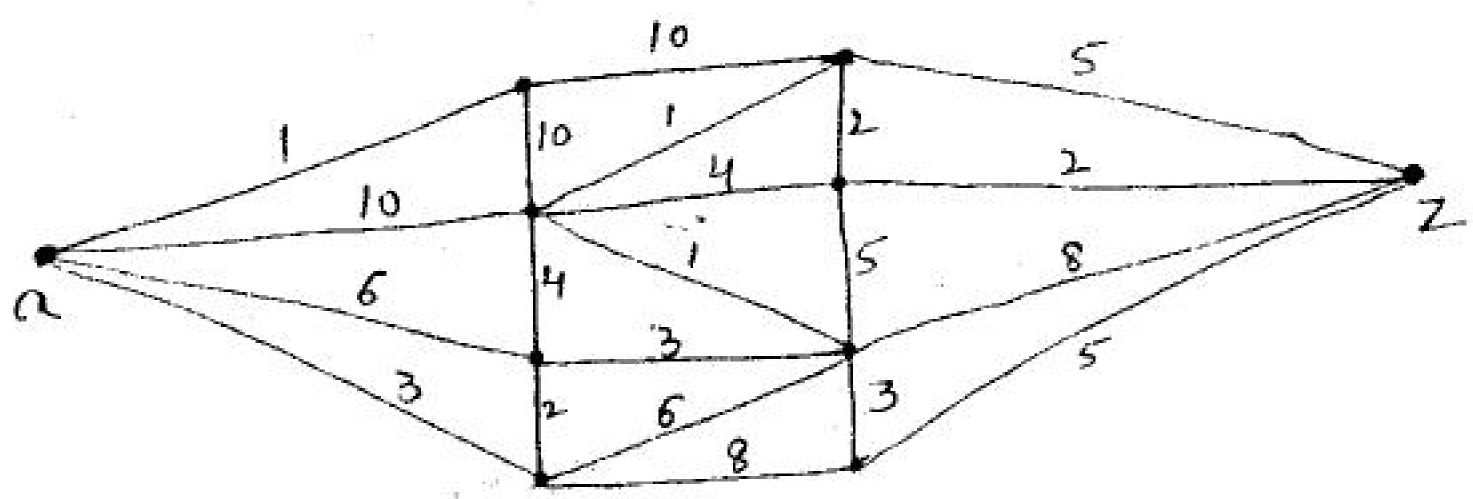
(b) Prove that in any non-trivial tree, there are at least two pendant vertices.

(c) If $F_2 = (V_2, E_2)$ is forest with $|V_2| = 62$ and $|E_2| = 51$; determine the number of trees in F_2. (7, 7, 6)

8. (a) A graph G has 31 edges, 8 vertices of degree 4 and all other vertices are of degree 3. Find the number of vertices in G.

(b) Explain Dijkstra's algorithm. Find the shortest path between a and z using the algorithm in the given graph; (5, 15)



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