Total No. of Questions-8] [Total No. of Printed Pages-4

B.E. IV Semester Examination

BE - IV/6(A)

214635

Computer Engineering

Course No. : MTH - 413

Discrete Mathematics

Time Allowed-3 Hours *Maximum Marks-100*

Note: Attempt any **five** questions selecting at least **Two** question from each section. Use of calculator is allowed.

Section - A

1. a) Define mathematical induction. Prove that $2^n \times 2^n - 1$ is divisible by 3 $\forall n \geq 1$.

   b) Determine the no. of integers between 1 to 250 that are divisible by any one of the integers 2,3,5 and 7.

   c) Define a countable set. Show that the set of integers **Z** is countable. **(6,7,7)**

2. a) Prove that the argument p,q $\vdash (p \vee r) \wedge q$ is valid without using truth table.

   b) Let R be a binary relation on the set of all positive integers S.t $R = \{(a,b) / a = b^2\}$. Is R reflexive or Symm. or Antisymmetric or Transitive or on equivalence relation or partial order relation ?

   c) Prove that a function $f : R \to R$, defined by $f(x) = x^3$ is one - one onto. **(7,7,6)**

3. a) Show that any two left cosets of H in G are either identical or disjoint.

b) Define cyclic group. Prove that every subgroup of a cyclic group is cyclic

c) Let $(\{x,y\},.)$ be a semi gp. Where $x.x = y$ show that $y.y = y$. **(7,7,6)**

4. a) Show that union of two subgroups H_1 & H_2 of a gp. G is a subgp. of G iff $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$.

b) Define field. Let $Q(\sqrt{2}) = \{a + b\sqrt{2} : a, b \in Q\}$.

Prove that $(Q\sqrt{2}; +;.)$ is a field where +,. Stand for addition and multiplication.

c) Define homomorphism of groups. Show that each of the following mappings is a homomorphism.

i) $f_1 : (C,+) \to (R,+)$ where $f_1(x+iy) = x$

ii) $f_2 : (C,+) \to (C,+)$ where $f_2(x+iy) = iy$

(7,7,6)

Section - B

5. a) Define the following terms with suitable example:

i) Directed graph

ii) Weighted graph

iii) Loop

iv) Complete graph

v) Multigraph.

b) Prove that a graph G has Eulter path iff it has either no vertex of odd degree or two vertices of odd degree.

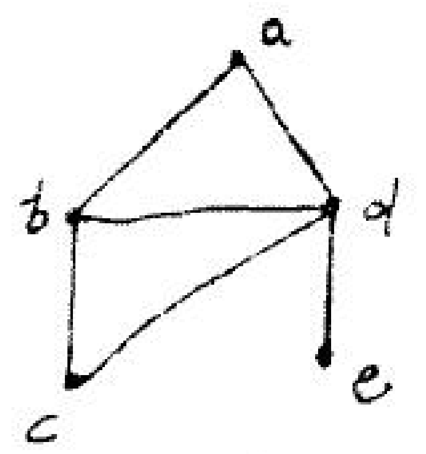
(10,10)

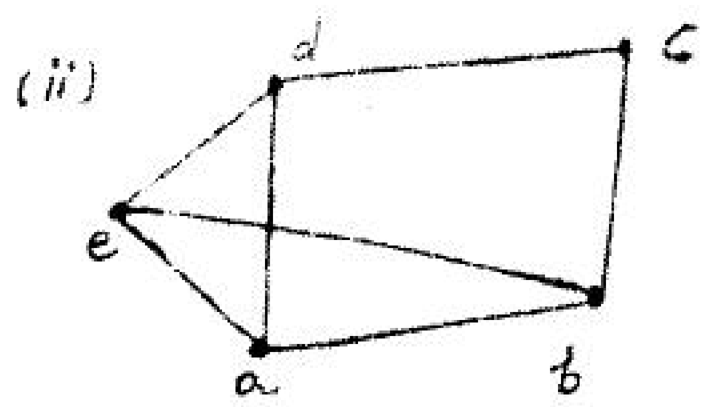
6. a) Prove that the no. of edges in a complete graph with n vertices is $n(n-1)/2$.

b) Is there exist a non simple graph G with deg. seq. (1,1,3,3,3,4,6,7)? Justify your answer.

c) Draw the complements of the following graphs:

(i)



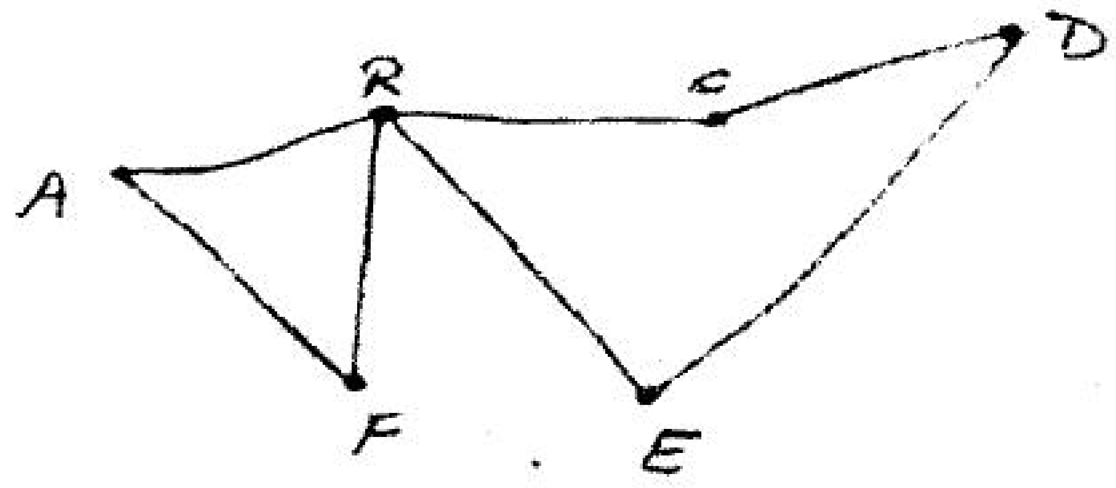


(7,6,7)

7. a) Explain chinese postman problem.

b) Prove that a tree with n-vertices has n-1 edges.

c) Define spanning tree. Generate a spanning tree for:



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(6,7,7)

8. a) A graph G has 31 edges, 8 vertices of degree 4 and all other vertices are of degree 3. Find the no. of vertices in G.

b) Explain Dijkstra's algorithm. Apply this algorithm to determine a shortest path between a & z in the graph given below. **(5,15)**