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Total No. of Questions – 8] [Total No. of Printed Pages – 4

BE-I/12(A)

231055

ENGINEERING MATHEMATICS – I COURSE NO. MTH – 101

Time Allowed: 3 Hours *Maximum Marks: 100*

Note: Attempt ***five questions*** in all selecting at least two questions from each Section. Each question carries 20 ***marks***. Use of calculator is allowed.

Section – A

1. (a) Find the radius of curvature at any point of the curve:

$$x = a(t + \sin t),\ y = a(1 + \cos t).$$

(b) Trace the curve: $3ay^2 = x(x-a)^2$, where $a > 0$. Also find the length of a loop of this curve.

(c) Find all the asymptotes of $x^3 + y^3 - 3axy = 0$ (6, 8, 6)

2. (a) Find the value of $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2}$

Where $u(x, y) = e^{-2xy} . \sin(x^2 - y^2)$

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(b) Examine the function:

$$f = (x - y)^2 + x^3 - y^3 + x^5$$

for extreme values at the origin.

(c) Evaluate $\int_0^{\pi/2} \sqrt{\tan x}\, dx \times \int_0^{\pi/2} \sqrt{\cot x}\, dx$ (7, 7, 6)

3. (a) Find $y_n(0)$, where $y(x) = \sin(m \sin^{-1} x)$

(b) Find the surface area of the solid generated by the revolution of the cardioids $r = a(1 - \cos\theta)$ about the initial line.

(c) Evaluate $\iint_R (1 - x^2 - y^2)\, dx\, dy$ over the triangular region R whose vertices are (1, 0), (2, 0) and (2, 2). (7, 7, 6)

4. (a) Find the position and nature of double points on the curve:

$$y(y - 6) = x^2(x - 2)^3 - 9$$

(b) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\, dz\, dy\, dx$

(c) Expand $f(x, y) = \sin x \cdot \cos y$ as a series about $(\pi/2, 0)$ up to 4^{th} degree terms. (7, 7, 6)

Section – B

5. (a) Express $\tan^{-1} z$ into real and imaginary parts.

(b) Sum the following series to infinity:

$\sin^2\alpha - \frac{1}{2}\sin 2\alpha \sin^2\alpha + 1/3 \sin 3\alpha \sin^3\alpha - \cdots\cdots$

(c) Prove that $\tan\left(i \log \frac{a-ib}{a+ib}\right) = \frac{2ab}{a^2-b^2}$ (7, 7, 6)

6. Solve the following differential equations:

(a) $(x^2+y^2+2x)\,dx + 2y\,dy = 0$

(b) $(1-x^2)\frac{dy}{dx} + 2xy = x.\sqrt{(1-x^2)}$

(c) $(1+x^2).\left(\frac{dy}{dx} - 4x^2\cos^2 y\right) + x.\sin 2y = 0$ (7, 7, 6)

7. Solve the following differential equations:

(a) $x^2y'' + xy' - y = \log x.\cos(\log x)$

(b) $(D^2+n^2)\,y = \cot nx$

(c) $(D^2+1)\,y = \sin x.\sin 2x$ (7, 7, 6)

8\. (a) Find the equation of the right circular cylinder of radius 2 and whose axis is the line:

$$\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$$

(b) Prove that the equation:

$$2x^2 + 2y^2 + 7z^2 - 10yz - 10xz + 2x + 2y + 26z - 17 = 0$$

represents a cone whose vertex is (2, 2, 1).

(c) Find the equation of the sphere through the circle:

$x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

(7, 7, 6)

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$\int 2^x$