otal No. of Questions - 13 Total printed pages: 05

B.E. I Semester Examination

BE-I/11(A)

229868

Engineering Mathematics

Course Code : BSC-101

Time Allowed: Three Hours *Maximum Marks – 100*

Note:

(i) Attempt Q.No. 1, which is compulsory and it consists of short answer questions of total 10 marks.

(ii) From each unit, attempt one question of 15 marks each (Total marks : $15\times6=90$)

1. Verify whether the following statements are true or false. $1\times10=10$

   (i) The curve $y^2=x^3$ has a node at the origin.

   (ii) The curve $r=a\cos2\theta$ has two leaves.

   (iii) $\sin z=\sin x.\cosh y+i\cos x.\sinh y$.

   (iv) $\sin x$ and $\sin 2x$ are independent solutions of $(D^2+1)y=0$

   (v) $(x^2+y^2+2x)\,dx+2y\,dy=0$ is an exact differential equation.

   (vi) The function $z=x.\sin^{-1}\left(\frac{y}{x}\right)$ is homogeneous of degree 2.

   (vii) The volume of a sphere of radius 'a' is $\frac{4}{3}\pi a^3$.

[Turn Over

(viii) The value of $\int_0^{\pi/2} \sin^7 x\,dx$ is $\frac{16}{35}$.

(ix) $\cosh(x+y) = \cosh x \,.\, \cosh y - \sinh x \,.\, \sinh y$.

(x) The angle between the vectors

$$4\hat{i} - 2\hat{j} + \hat{k} \text{ and } i + j - 2\hat{k} \text{ is } \frac{\pi}{2}$$

(xi) div $\vec{r} = 3$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

UNIT – I

2. (a) Find all the asymptotes of $x^3 + 3x^2y - xy^2 - 3y^3 + x^2 - 2xy - 3y^2 + 4x + 5 = 0$.

(b) Trace the curve : $r = a \sin 2\theta$.

(8,7)

OR

3. (a) Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$, where $u = \tan^{-1}\left(\frac{y}{x}\right)$

(b) Find the position and nature of double point on $a^2y^2 = a^2x^2 - 4x^3$.

(c) Find the radius of curvature at any point of the cardioid $r = a(1 - \cos\theta)$.

(5,5,5)

UNIT – II

(a) Verify Rolle's theorem for

$$f(x) = e^x (\sin x - \cos x) \text{ in } \left[\frac{\pi}{4}, \frac{5\pi}{4}\right].$$

(b) Examine the function $f = xy(a - x - y)$ for extreme values. (8,7)

OR

5. (a) Determine $\lim_{x \to 0}\left(\frac{\tan x}{x}\right)^{1/x^2}$

(b) Expand $f(x, y) = e^x \log(1 + y)$ as a series in powers of x and y upto third degree terms.

(c) State and prove mean value theorem. (5,5,5)

UNIT – III

6. (a) Find the length of the loop of the curve : $3ay^2 = x(x-a)^2$

(b) Evaluate $\iint_R (x^2 + y^2)\,dxdy$ over the annular region R lying between $x^2 + y^2 = 4$ and $x^2 + y^2 = 9$. (8, 7)

OR

7. (a) Find the volume of the solid generated by revolving area about the x-axis, of the area included between $y^2 = x^3$ and $x^2 = y^3$.

(b) Evaluate $\int_0^\infty x^3 . e^{-\sqrt{x}} dx$.

(c) Evaluate $\int_{-1}^{1} (1 + x)^{3/2} (1 - x)^{7/2} dx$ by using beta/gamma functions. (5,5,5)

UNIT – IV

8. (a) Verify Green's theorem in the xy - plane for $\oint_C (2xy - x^2)$ dx + $(x^2 + y^2)$ dy where C is the boundary of the region enclosed by curves $y = x^2$ and $x = y^2$.

(b) Evaluate $\oint_C \vec{F}.d\vec{r}$ around a triangle in the xy-plane with vertices at (0, 0), (3, 0) and (3, 2) taken in counter clockwise direction, where

$$\overline{F} = (2x - y + 4)\hat{i} + (5y + 3x - 6)\hat{j}.$$

(8,7)

OR

9. (a) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 - z = 3$ at the point $(2, -1, 2)$.

(b) Show that $\text{div}(\text{curl}\,\vec{F}) = 0$.

(c) Show that $\oiint_S \nabla r^2 . dS = 6V$, where $r = |\vec{r}|$, $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and S is a closed surface enclosing a volume V.

(5,5,5)

UNIT – V

10. (a) Express $\tan^{-1} Z$ into real and imaginary parts.

(b) If $e^{i\theta} = \sinh(x + iy)$, show that

$$\text{Sinh}^4 x = \cos^2\theta = \cos^4 y.$$

(8,7)

OR

11. (a) Show that $\tan\left[i \log\left(\frac{a - ib}{a + ib}\right)\right] = \frac{2ab}{a^2 - b^2}$

(b) Show that $\sin(\log i^i) = -1$.

(c) Sum the series to infinity. (5,5,5)

$$1 + x\cos\theta + \frac{x^2}{2!}\cos 2\theta + \frac{x^3}{3!}\cos 3\theta + \ldots$$

UNIT – VI

12. (a) Solve:

$$x^2 y'' + 7xy' + 5y = 4(x^{-1} + x^{-2})$$

(b) Solve:

$$y'' - y = \frac{2}{1 + e^x}$$ (8,7)

OR

13. Solve:

(a) $(D^3 - D^2 - 6D)\, y = x^2$.

(b) $y(e^x + 2xy)\, dx - e^x dy = 0$

(c) $\dfrac{dy}{dx} + 4xy + xy^3 = 0$ (5,5,5)

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