Total No. Of Questions-8] [Total No. of Printed Pages-4

B.E. II Semester Examination

BE-II/6(A)

214155

ENGG. MATHS. - II

Course No.: MTH-201

Time Allowed- 3Hours *Maximum Marks-100*

Note:- Attempt **FIVE** questions in all, selecting **at least two** questions from **each section**. All questions carry **equal** marks.Use of calculator is allowed

Section-A

1. a) State and prove p-series test

b) Discuss the convergence or divergence of the series:

$$1+\frac{1}{2}x+\frac{1.3}{2.4}x^2+\frac{1.3.5}{2.4.6}x^3+....$$

c) Discuss the convergence or divergence of the series

$$\frac{x}{1.3}+\frac{x^2}{3.5}+\frac{x^3}{5.7}+....$$ **(6,7,7)**

2. a) Draw the graph of the function

$$f(x)=\begin{cases} a+x, & -a\le x<0 \\ a-x, & 0\le x\le a \end{cases}$$

Is it odd or even? Also expand it as fourier series in (-a,a)

2500 [Turn Over

b) Using Fourier cosine series of f(x), find the value of

$$\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}\text{ , where } f(x)=\begin{cases} x \text{ , } & 0\le x< c/2 \\ c-x\text{ , } & c/2\le x\le c\end{cases}$$

c) Using parseval's identity for fourier series of $f(x)=x^2$ in $(-\pi,\pi)$, find the value of $\sum_{n=1}^{\infty}\frac{1}{n^4}$ **(6,7,7)**

3. a) Determine ordinary or singular points (Regular and irregular) of the differential equation.

$$x^2(x-1)(x+2)y''-x(1-x)y+2x(x+2)y=0$$

b) Find the indicial equation and recurrence relation of the differential equation $x^2y''+(x^2-3x)y'+3y=0$

c) Find the power series solution about $x=0$ of the differential equation: $(1-x^2)y''-xy'+p^2y=0$, where p is a constant. **(6,7,7)**

4. a) Derive fourier sine series of $f(x)=x^2-x$ in $(0,1)$

b) Find Frobenius series solution near $x=0$ of the differential equation: $2x^2y''-xy'+(1-x^2)y=0$

c) Find the interval of convergence of the series

$$x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\frac{x^5}{5}.....$$ **(6,7,7)**

Section-B

5. a) Obtain p.d.e by eliminating arbitrary functions f and g from the relation $z = f(2x-3y)+g(x+2y)$

b) Solve $(xz+y^2)p+(yz-2x^2)q+2xy+z^2=0$

c) Solve $q = yzp^2$ **(6,7,7)**

6. a) Solve $x^n p^n + y^n q^n = z^n,\ n \neq 1$

b) Solve $(D^3 - 7DD'^2 + 6D'^3)z = e^{x+y} + \sin(x+y)$

c) Solve $(D^2 + D'^2 - 2DD' - 3D + 3D' + 2)z = e^{2x-y}$

(6,7,7)

7. a) Find the solution of one dimensional heat equation

$\frac{\partial^2 u}{\partial x^2} = \frac{1}{k}.\frac{\partial u}{\partial t}$ under the conditions that

$$u(0,t)=0,\ \forall t$$
$$u(L,l)=0\ \ \forall t$$
$$and\, u(x,0)=x,\ \forall x$$

b) Find two non singular matrices P and Q such that PAQ is in normal form, where

$$A=\begin{bmatrix}-1 & 2 & 3 & 4\\ 4 & 3 & 2 & -1\\ 3 & 4 & 1 & 2\end{bmatrix}$$

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c) Test the system of equations for consistency, if possible, solve them (6,7,7)

$$3x + y + 2z = 3,\ 2x - 3y - z = -3,\ x + 2y + z = 4$$

8. a) Define rank of a matrix. Also reduce the matrix A to normal form and find its rank where

$$A = \begin{bmatrix} 24 & 19 & 36 & -38 \\ 49 & 40 & 73 & -80 \\ 73 & 59 & 98 & -118 \\ 47 & 36 & 71 & -68 \end{bmatrix}$$

b) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

Also find eigen values of A^5

c) Find all the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$$

(6,7,7)