Total No. of Questions-8] [Total No. of Printed Pages-3

B.E. I Semester Examination

BE-I/12(A)

227922

MATHEMATICS - I

Course No. MTH-101

Time Allowed- 3Hours *Maximum Marks-100*

Note : Attempt **five** questions in all, selecting at least **two** from each section. All carry **equal** marks. Use of calculator is allowed.

Section - A

1. a) Given $y = (\sin^{-1} x)^2$, find the value of its n^{th} - derivative at $x=0$.

b) Find extreme value of $x^2+y^2+z^2$ when $ax+by+cz=p$.

c) Find the radius of curvature at origin of the curve:

$$2x^3 + 4x^2y + xy^2 + 5y^3 + x^2 - 2xy + y^2 - 4x = 0.$$

(7,7,6)

2. a) Find all asymptotes of the curve given by

$$(2x-3y+1)^2(x+y) = 8x - 2y + 9.$$

b) If $x^2 + y^2 + z^2 = e^{2u}$, then show that

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = e^{-2u}$$

c) Evaluate $\int_0^{\pi/2} \log(\sin\theta)\, d\theta$. **(7,7,6)**

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3. a) Show that

$$\int_0^t erf_c(ax)dx = t\ erf_c(at) - \frac{1}{a\sqrt{\pi}}\left(e^{-a^2t^2} - 1\right).$$

b) Find the length of the arc of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant.

c) Find the area of loop of the curve $a^2y^2 = x^3(2a - x)$ above x-axis. **(7,7,6)**

4. a) Find the surface area of the solid generated by the revolution of the loops of the curve $r^2 = a^2 \cos 2\theta$ about the initial line.

b) Evaluate $\iint r \sin\theta dr\ d\theta$ over the cardioid $r = a(1 - \cos\theta)$ above the initial line.

c) Evaluate $\iiint (x + y + z)\,dx\,dy\,dz$ over the tetrahedron bounded by the planes $x = 0, y = 0, z = 0$ and $x+y+z=1$.

(7,7,6)

Section - B

5. a) If $\tan(x + iy) = \cos\alpha + i\sin\alpha$, prove that

$$x = \frac{\pi}{4}(2n+1)\ and\ \ y = \frac{1}{4}\log\tan\left(\frac{\pi}{4} + \frac{\alpha}{2}\right).$$

b) Sum the series to infinity.

$$1 + x\cosh\alpha + x^2\cosh 2\alpha + x^3\cosh 3\alpha + \ldots\ldots$$

c) Show that

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$ and
$\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$

6. Solve the following differential equations :

a) $\left(1+e^{x/y}\right)dx + e^{x/y}\left(1-\frac{x}{y}\right)dy = 0$

b) $\left(1+x^2\right)\left(\frac{dy}{dx} - 4x^2\cos^2 y\right) + x\ sin2y = 0$

c) $\left(D^2+5\right)y = x\sin x.$ (7,7,6)

7. Solve the following differential equations :

a) $y'' - 2y' + y = \frac{e^x}{x}$

b) $x^3y''' + 2x^2y'' + 2y = 20(x+x^{-1})$

c) $\left(D^2+2\right)y = x^2e^{3x} + e^x\cos 2x.$ (7,7,6)

8. a) Find the equations of the lines in which the plane $2x+y-z=0$ cuts the cone $4x^2 - y^2 + 3z^2 = 0$. Also find the angle between the lines.

b) Find the equation of the right circular cylinder whose axis is $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$ and radius 2.

c) Find the angle of intersection of the spheres $x^2+y^2+z^2+6y+2z+8=0$ and $x^2+y^2+z^2+6x+8y+4z+20=0.$ (7,7