BE-I/6(B)

227293

MATHEMATICS-I

Course No. MTH - 101

Time Allowed-3Hours *Maximum Marks-100*

Note: Attempt **five** questions in all, selecting atleast **two** questions from each section. All carry **equal** marks. Use of calculator is allowed.

Section - A

1. a) If $y = x^{n-1} \log x$, prove that $xy_n = (n-1)!$.

b) Find the value of the expression $x^2 u_{xx} + 2xy u_{xy} + y^2 u_{yy}$,

where $u = \sec^{-1}\left(\frac{x^2 - y^2}{x^5 + y^5}\right)$.

c) Find the radius of curvature at any point of the curve $y^2 = 4ax$. **(7,7,6)**

2. a) Find all the asymptotes of the curve $(x + 9y + 2)(x + y)^2 = x + 3y - 1$.

b) Trace the curve given by $x = a(t + \sin t),\ y = a(1 + \cos t)$.

c) Examine the function $x^2 + y^2 + z^2$ for extreme values given that $xyz = k^3$. **(7,7,6)**

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3. a) Show that $\beta(m,n) = \frac{\Gamma(m)\,\Gamma(n)}{\Gamma(m+n)}$, where the

b) Find the surface of the solid generated by the revolution of the ellipse $x^2 + 4y^2 = 16$ about its minor axis.

c) Find the area of a loop of the curve $r^2 = a^2 \cos 2\theta$ **(7,7,6)**

4. a) Find the total length of the curve $x^2(a^2 - x^2) = 8a^2y^2$.

b) Change the order of integration in $\int_0^2 \int_y^2 \frac{x\,dx\,dy}{x^2 + y^2}$ and hence evaluate it.

c) $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} xyz\,dz\,dy\,dx$. **(7,7,6)**

Section - B

5. a) Prove that $\log\left(\frac{a+ib}{a-ib}\right) = 2i\tan^{-1}\left(\frac{b}{a}\right)$.

b) Sum of the following series to infinity:

$$1 + x\cos\theta + \frac{x^2}{2!}\cos 2\theta + \frac{x^3}{3!}\cos 3\theta + \ldots\ldots$$

c) Express the function $\tan z$ into real and imaginary parts. **(7,7,6)**

6. Solve the following differential equations:

a) $(y^2 e^{xy^2} + 4x^3)\,dx + (2xy\,e^{xy^2} - 3y^2)\,dy = 0$

. . . .

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a) $(y^2 e^{xy^2} + 4x^3)\,dx + (2xy\,e^{xy^2} - 3y^2)\,dy = 0$

b) $x(1+x^2)\frac{dy}{dx} = y(1-x^2) + x^2 \log x$

c) $(1+e^{y/x})\cos x\,dx + e^{y/x}\sin x\,dy = 0$ (7,7,

7. a) Solve: $x^2y'' + 2xy' - 20y = (x+1)^2$.

b) Using method of variation of parameters solve $y'' - 2y' + y = e^x \log x$.

c) Solve: $(D^2 - 2D + 1)y = x\sin x$. **(7,7,6)**

8. a) Find the equation of the tangent plane to the sphere $3(x^2 + y^2 + z^2) - 2x - 3y - 4z - 22 = 0$ at the point (1,2,3).

b) Prove that the equation $4x^2 - y^2 + 2z^2 - 3yz + 2xy + 12x - 11y + 6z + 4 = 0$ represents a cone whose vertex is (-1,-2,-3).

c) Find the equation of the right circular cylinder whose guiding circle is $x^2 + y^2 + z^2 = 9, x - y + z = 3$. **(7,7,6)**