Total No. of Questions-8] [Total No. of Printed Pages-4

B.E. I Semester Examination

BE-I/12(A)

227991

MATHEMATICS-I

Course No. MTH-101

Time Allowed-3 Hours *Maximum Marks-100*

Note :- Attempt **five** questions in all, selecting atleast **two** questions from each section. All carry **equal** marks. Use of calculator is allowed.

Section - A

1. a) If $y\sqrt{1-x^2} = \sin^{-1} x$, prove that

$$(1-x^2)y_{n+1} - (2n+1)xy_n - n^2 y_{n-1} = 0$$ and hence evaluate $y_n(0)$.

b) Find the value of the expression

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2},$$

where $u = \cot^{-1}\left(\frac{x^2 + y^2}{x^{1/3} + y^{1/3}}\right)$

[Turn Over

c) Find all the asymptotes of the curve:

$2x^3 - 2xy^2 - x^2y + y^3 - 4x^2 + 8xy - 4x + 1 = 0$. (7,7,6)

2. a) Examine the function

$f(x, y) = \sin x + \sin y + \sin(x + y)$ for extreme values.

b) Find the radius of curvature at any point of the curve

$r^n = a^n \cos n\theta$.

c) Find the position and nature of double points on the curve

$x^3 + y^3 = 35xy$. (7,7,6)

3. a) Determine the area common to the two parabolas $x^2 = 12y$ and $y^2 = 12x$

b) Determine the length of the loop of the curve $3ay^2 = x(x-a)^2$.

c) Show that $\beta(m,n) = \beta(m,n+1) + \beta(m+1,n)$ (7,7,6)

4. a) Evaluate the integral $\iint_R x^2 y^2 \, dx \, dy$ over the region bounded by the unit circle.

a) Evaluate the integral $\iiint (x+y+z) \, dx \, dy \, dz$ over the tetrahedron bounded by the planes $x=0$, $y=0$, $z=0$ and $x+y+z=1$

c) Using gamma function, evaluate $\int_0^1 (x \log x)^6 \, dx$. (7,7,6)

Section - B

5. a) Express $\tan^{-1}(z)$ into real and imaginary parts.

b) If $y = \log \tan\left(\frac{\pi}{4} + \frac{x}{2}\right)$, then show that $\cosh y \cos x$

c) Prove that the equation

$7x^2 + 2y^2 + 2z^2 - 10xz + 10xy + 26x - 2y + 2z - 17 = 0$

represents a cone whose vertex is $(1, -2, 2)$. (7,7,6)

6. a) Find the equation of the right circular cylinder whose axis is $x = 2y = -z$ and radius is 4.

b) Solve:

$$\left[y\left(1 + \frac{1}{x}\right) + \cos y\right]dx + \left[x + \log x - x \sin y\right]dy = 0$$

c) Solve: $\left(xy^2 - e^{1/x^3}\right)dx = x^2 y\, dy$ (7,7,6)

7. a) Using variation of parameters technique, solve the differential equation: $y'' + y = \frac{1}{1 + \sin x}$

b) Solve: $x^2 y'' + 3xy' + 5y = 4\left(x^{-1} + x^{-2}\right)$.

c) Solve: $\left(D^4 + 1\right)y = x^2 + \sin x \cos x$. (7,7,6)

a) Sum to infinity the following series:

$$\cos x \sin x - \frac{1}{2}\cos 2x \sin^2 x + \frac{1}{3}\cos 3x \sin^3 x - \ldots.$$

b) Solve . $\left(D^2 - 2D + 1\right)y = x \sin x$.

c) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9, 2x + 3y + 4z = 5$ and the point (1,2,3) (7,7,6)